

Stock Prices, Exchange Rates and Monetary Policy*

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Abstract

This paper attempts to model the relationship between monetary policy and financial asset prices. We develop an aggregative model under forward-looking rational expectations to analyse the optimal monetary policy response to stock prices and exchange rates shocks. We first demonstrate that a model ignoring the impact of equity prices and exchange rates on aggregate demand leads to an overestimation of the optimal policy response to standard shocks. Second, we clearly point out that a correct assessment of the relation between optimal monetary policy and either equity prices or exchange rates necessitates a model including both kinds of financial prices simultaneously. Third, we show how these interactions between financial asset prices and monetary policy are affected by a particular form of coordination between monetary policy and fiscal policy, arising from a public debt solvency constraint.

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1. Introduction

This paper attempts to clarify some specific issues about the relation between financial asset prices, such as equity prices and exchange rates, and monetary policy. These issues involve the optimal monetary policy response to exchange rate and equity price movements, and the feedback effect of monetary policy on the determination of these financial asset prices.

Our paper contributes to the existing literature several ways. First, we demonstrate that a model ignoring the impact of equity prices and exchange rates on aggregate demand leads to an overestimation of the optimal policy response to standard demand shocks. Second, we clearly point out that a correct assesment of the relation between optimal monetary policy and either equity prices or exchange rates necessitates a model including both kinds of financial prices simultaneously. Third, we show how these interactions between financial asset prices and monetary policy are affected by a particular form of coordination between monetary policy and fiscal policy, arising from a public debt solvency constraint.

The view taken in this paper is that the main task of Central Banks is to control inflation, which heavily depends on the output gap. This is the reason why Central Banks try to stabilize aggregate demand through a fine tuning of short term interest rates. Interest rates affect aggregate demand both directly and indirectly. Aggregate demand directly depends on interest rates through their effects on investment and consumption expenditures. But interest rates also affect aggregate demand indirectly, through their effects on exchange rates and equity prices. It is indeed well established that exchange rate movements influence net exports. It is also widely recognized that equity prices affect private consumption and corporate investment. High equity prices increase consumers wealth and boost consumption (see Ludvigson and Steindel, 1999 for an empirical assessment). High equity prices also increase Tobin's Q and improve firms ability to raise funds in the markets or to borrow from the banks. Therefore, financial asset prices such as exchange rates and equity prices are an essential part of the monetary policy transmission mechanism. Two aspects of this issue deserve particluar attention. First, even when monetary policy reacts to unexpected aggregate demand fluctuations which are independent of any initial movement of equity prices or exchange rates, these financial asset prices ultimately move because they are directly affected by the interest rate reaction decided by the Central Bank. Second, when deciding of how much the interest

rate has to be changed in response to some shock of aggregate demand and in order to achieve a particular inflation objective monetary authorities have to take account of the induced movements in financial asset prices which in turn affect aggregate demand and inflation. This is one of the reasons why some Central Banks adjust their policy by relying on a Monetary Conditions Indicator which generally contains at least a short-term interest rate and an exchange rate, as an operating target. It is our view that for reasons outlined above, equity prices should be incorporated in such a Monetary Conditions Indicator. In this paper we derive such an augmented MCI from optimal central bank behaviour.

In our paper the analysis is conducted within the framework of an aggregative model which is a variant of a standard Barro-Gordon (1983) model, with supply and demand shocks. We add to the model some arbitrage equations describing the stochastic dynamics of asset prices. Each of these equations contains a systematic part and a stochastic unsystematic part, modelled as a random shock. Solving this model yields, under the optimal monetary policy rule described by the MCI relation, the reactions of interest rates, exchange rates and equity prices to the various shocks affecting the economy, including shocks to financial asset prices. The interpretation of these shocks deserves some comments. Volatility is a normal attribute of financial asset prices since their fluctuations are theoretically governed by revisions of the expected future values of their determinants. But financial prices may also fluctuate in response to exogenous variations of risk premia. Insofar as they affect the output gap and the inflation outlook these shocks influence monetary policy.

We do not address the question as to whether monetary policy should respond to bubbles which may lead equity prices and exchange rates levels to depart considerably from their theoretical values. It is often argued that equity prices and exchange rates sometimes display excessive volatility, which is not warranted by the evolution of their fundamental determinants according to economic theory. We concede that a concern about bubbles and asset price stabilization by monetary policy can be well motivated. Bernanke and Gertler (1999) explain that non fundamental factors sometimes underlie asset market volatility because of poor regulatory practice and imperfect rationality on the part of investors. The question often arises as to whether monetary authorities have to react to those exogenous fluctuations of financial asset prices that are not justified by their fundamentals. It is however true that is very difficult to decide whether a given change in asset value results from fundamental factors or not, as shown by Cogley (1999) and Bernanke and Gertler (1999).

Therefore unsystematic changes of financial prices are interpreted in this paper as stochastic changes of the risk premium. Moreover a correct assessment of the mechanism via which non-fundamental movements in asset prices may disrupt the economy necessitates a model whose assumptions differ from ours, see Bernanke and Gertler (1999). These authors anyway conclude that monetary authorities should not try to stabilize financial prices per se but should simply attempt to control inflation. As a by product such an inflation targeting policy may contribute to smooth the variance of financial prices.

To deal with some of the issues addressed in this paper, some pioneering work has been undertaken by Gerlach and Smets (1996) and Smets (1997), but the models in these papers involve only one asset price (whatever its interpretation). The optimal monetary policy response to exchange rate movements is modelled in a paper by Gerlach and Smets (1996). In a connected paper, Smets (1997) models the interaction between monetary policy and equity prices fluctuations. In this latter paper, Smets points out a close analytical similarity between modelling the role of equity prices and modelling the role of exchange rate. He therefore claims that his model may also be interpreted as dealing with the relation between monetary policy and exchange rates rather than equity prices. Our paper however demonstrates that it is necessarily to include both kinds of financial prices in the model in order to correctly assess their impact on monetary policy.

This paper extends previous analyses several ways. In the real world several kinds of asset prices simultaneously interact with monetary policy. This implies that a shock on one particular asset price affects other asset prices through the monetary policy response it implies. When monitoring its interest rate response to a shock on a particular asset price, the Central Bank has to take account of the induced effects on other asset prices and their feedback on the one which was initially affected. Our work attempts to clarify this interaction between intervention interest rates, equity prices and exchange rates. We show that a correct assesment of optimal monetary policy necessitates to model the role of both kinds of financial prices simultaneously. We explicitly introduce two assets: equities and exchange rates. In spite of the analytical complexity induced by this exercise, we think that this effort is justified by the above arguments. It may also be pointed out that the analytical similarity in modelling the role of equities and exchange rates is only partial. Contrary to exchange rates, equity prices are indeed directly affected by the expected value of some real variables. Exchange rates are only indirectly affected by expected real variables through

the induced variations of interest rates.

Another original feature of our base model is that we explicitly describe how wages and productivity, and therefore profitability, may affect financial asset prices and monetary policy. To achieve this objective we assume that real dividends depend on the wedge between real production and real wage costs.

A major contribution of our analysis is that we examine how the interaction between monetary policy and financial asset prices is affected by the introduction of fiscal policy in the model. We thus specify an extended model involving a particular form of coordination between monetary policy and fiscal policy. This coordination arises from a public debt solvency constraint. In most industrial countries, some form of real debt stabilisation requirement is binding (an example is provided by the European Stability Pact but similar objectives are observed in the United States and many other countries). Any contracyclical movement of monetary policy is accompanied by a similar contracyclical movement of fiscal policy. Our opinion is that any discussion about monetary policy issues has to take account of this point. We therefore take explicitly account of a public debt solvency constraint, inspired by Krichel, Livine and Pearlman (1996) and Van Aarle, Bovenberg and Raith (1995). This constraint establishes a strong link between fiscal and monetary policies, since monetary policy determines inflation and seigniorage. We derive how the optimal monetary policy response to shocks on financial asset prices is affected when we take account of the necessary adjustment of government expenditure. For clarity of exposition, and to highlight the specific features of the results implied by monetary and budgetary policy coordination, we first develop a base model without fiscal policy. Thereafter we extend the analysis to take account of fiscal policy.

We thus develop a model where aggregate demand depends on real equity prices and a real exchange rate. In this model, the Central Bank is assumed to have an objective function depending on output deviation from a natural level and domestic output price inflation deviation from a target level, as described above. Following Barro and Broadbent (1997), Gerlach and Smets (1996) and Smets (1997), our paper assumes that monetary authorities minimize a quadratic loss function depending on deviations of output from its potential or natural level and deviations of inflation from its target level. This objective implies that the Central Bank has no incentive to surprise the private sector with inflation, even in response to supply shocks, contrary to what happens in Barro and Gordon (1983).

To derive the solutions we use a novel methodology, introduced by Broadbent

and Barro (1997), which avoids some of the cumbersome calculation associated with solving rational expectations models with an explicit goal for the monetary authority. However, since our model contains endogenous financial asset prices, the analytical derivation of the solutions requires some additional steps. We have to solve a system of two simultaneous first order difference equations which we transform into two independent second order difference equations.

The structure of the paper is organised as follows. This introduction is the first section of the paper. Section 2 develops the base model. Section 3 derives the main properties of optimal monetary policy under output gap stabilisation and inflation rate targeting. These properties allow us to derive a monetary conditions index which describes optimal monetary policy in the context of our base model. Section 4 solves the model, and the outcome is discussed at length. Section 5 develops an extension of the model: fiscal policy is introduced under a binding debt solvency constraint. Section 6 concludes. The paper involves some cumbersome calculations but we focus on economic analysis, so that most analytical derivations are presented in appendices.

2. The base model

2.1. Description of the behaviour of private agents

We develop a simple aggregative model under rational expectations, to analyse the optimal monetary policy response to stock prices and exchange rates shocks. The log of aggregate demand y_t^d is assumed to depend negatively on the expected real interest rate, positively on the log of a real stock price l_t , and negatively on the log of the real exchange rate of the domestic currency in terms of the foreign currency:

$$y_t^d = \alpha - \beta(R_t - E_t\pi_{t+1}) + \gamma l_t - \mu(e_t + p_t - p_t^*) + \varepsilon_t^d \quad (2.1)$$

In this equation R_t denotes the nominal interest rate and π_{t+1} is the inflation rate on date $t+1$, so that $R_t - E_t\pi_{t+1}$ is the expected real interest rate since E_t is the expectation operator conditioned on information available on date t . The log of the real exchange rate is given by $e_t + p_t - p_t^*$ where e_t is the log of the nominal exchange rate while p_t and p_t^* denote the logs of domestic and foreign prices respectively. The exchange rate e_t is the price of the domestic currency in terms of the foreign currency. The date t inflation rate is simply

$$\pi_t = p_t - p_{t-1} \quad (2.2)$$

The log of date t output, y_t^s , differs from its natural or flexible-price equilibrium level, \bar{y} , by an amount proportional to the wedge between realised and expected prices (the price forecast error):

$$y_t^s = \bar{y} + \theta(p_t - E_{t-1}p_t) + \varepsilon_t^s \quad (2.3)$$

where ε_t^s denotes a supply shock (for example random fluctuations in the productivity of capital, or stochastic shifts in the labour supply curve) while \bar{y} is a constant. Such a supply function is usual in this kind of literature (see for example Broadbent and Barro, 1997) and can be rationalised by incomplete information about current prices or wages (Lucas, 1973 or Barro, 1976). Eq. 2.3 then reflects the maximizing behaviour of private agents on localized markets where they have only partial information about contemporaneous nominal aggregates. In these settings θ depends on the relative variances for general and market-specific shocks. Such a supply function can also be motivated by some forms of nominal labour contracts (see Gray, 1976 or Fischer, 1977). Supply is thus governed by innovations in aggregate producer prices and some exogenous stochastic process. Note also that $p_t - E_{t-1}p_t$ is equal to $\pi_t - E_{t-1}\pi_t$ so that aggregate supply increases with surprises in the inflation rate.

Domestic prices adjust to equal supply and demand. Therefore production y_t satisfies the following equilibrium condition:

$$y_t^s = y_t^d = y_t \quad (2.4)$$

The real stock price l_t follows an arbitrage condition according to which the expected real return on equities must be equal to the expected real riskless interest rate plus a time-varying risk premium ε_t^l . A log-linear approximation of this arbitrage condition, derived in appendix 1, is given by :

$$R_t - E_t\pi_{t+1} + \varepsilon_t^l = \rho E_t l_{t+1} + (1 - \rho)E_t d_{t+1} - l_t \quad (2.5)$$

where d_{t+1} denotes the log of real dividends on date $t+1$. This log-linear approximation is standard, see for example Campbell, Lo and MacKinlay (1997, page 261) and the references there quoted¹. The right hand side of this equation

¹In this context the nominal stock price is determined by a dividend discount model where the discount rate, or required return on equity, is the nominal interest rate R_t on non risky assets augmented by a risk premium ε_t^l which fluctuates randomly.

denotes the expected real return on equities, which is a convex combination of the expected capital gain and the expected dividend yield. Note that we do not interpret ε_t^l as a bubble which would represent the effects of self-fulfilling forecasts on securities markets. Therefore ε_t^l does not necessarily reflect variations in asset prices that are unrelated to fundamentals. We prefer to consider ε_t^l as a random fluctuation of the risk premium due to stochastic exogenous changes in investors risk aversion. To introduce a bubble in asset prices is irrelevant given the purposes of this paper because a bubble is not easily identified. Cogley (1999) convincingly argues that identification of asset prices bubbles requires more knowledge about asset price fundamentals than central banks possess. Bernanke and Gertler (1999) show that even when a bubble is present, the market price can still be expressed as a discounted stream of cash flows, though with a discount rate that differs from the fundamental rate. In order to capture the effects of labour productivity and wage formation on profitability, we assume that date $t+1$ real dividends are equal to the wedge between real production and real wage costs on date t . A log-linear approximation of this hypothesis is given by

$$d_{t+1} = y_t - \tau - \lambda w_t \quad (2.6)$$

where w_t denotes the log of the real wage rate on date t . This approximation is derived in appendix 2, where it is shown that τ depends on labour productivity. The ex ante real stock price is given by

$$l_t = L_t - E_t p_t \quad (2.7)$$

where L_t is the log of the nominal stock price and $E_t p_t$ denotes the perceived price level, since p_t is not observed on date t . Let lw_t be the log of the nominal wage rate. The ex ante real wage rate is defined by

$$w_t = lw_t - E_t p_t \quad (2.8)$$

while private agents commit themselves to nominal wage contracts one period before the contracts come into effect. Since they care about real wages, wage setters form expectations on next period's inflation, $E_{t-1}\pi_t$, to know what nominal wage to commit to in advance. If workers always commit to a wage that sets expected output at its natural level, nominal wage contracts satisfy $lw_t = lw_{t-1} + E_{t-1}\pi_t$. If forecasts turn out to be wrong, real wages may deviate from their market clearing level, and output may differ from its natural level.

These assumptions are consistent with the hypotheses underlying eq. 2.3². We also assume that, after supply decisions have been made³, nominal wages may differ from contracts by some wage shock ε_t^w , so that

$$lw_t = lw_{t-1} + E_{t-1}\pi_t + \varepsilon_t^w \quad (2.9)$$

Nominal exchange rates follow a standard uncovered interest rates parity condition, according to which the cross-country interest rate spread must reflect the anticipated change of the bilateral exchange rate :

$$R_t = R_t^* - E_t e_{t+1} + e_t + \varepsilon_t^x \quad (2.10)$$

where R_t^* denotes the foreign interest rate and ε_t^x is a time-varying risk premium arising from speculative exchange market pressures.

In our model describing the functioning of the economy, the variables p , y , w , l , L , d , e , π , and lw are endogenous, while R , R^* and p_t^* are exogenous. However R is set by the Central Bank, so that it is sufficient to add an equation describing optimal monetary policy to make R endogenous. Before doing this we detail our hypotheses about the informational structure in the economy, and we derive some implications of the model concerning the equilibrium price level.

2.2. Informational structure

Following Broadbent and Barro (1997) we assume that the aggregate price and output levels cannot be observable contemporaneously. They are only observed

²Eq. (1.3), rewritten as $p_t = E_{t-1}p_t + \frac{1}{\theta}(y_t^s - \bar{y} - \varepsilon_t^s)$, may also be derived by assuming that prices are set as a mark-up over wages and that wages are set one period in advance, see Canzoneri and Henderson (1991). The mark-up increases with the output gap. This interpretation is chosen by Smets (1997).

³Nominal wage contracts are set on date $t-1$, on the basis of expected inflation. On date t , given these nominal wage contracts, each firm i observes its own selling price p_{it} (but not the average price p_t) and faces a real wage contract level which may differ from the previously expected one. Given this real wage contract level, the output of each firm is determined, leading to the well known aggregate supply function (1.3). After production decisions have been made, labour market shocks may imply some departure of nominal wages from their contract levels, thereby affecting profitability and dividends. This hypothesis about the timing of the wage shock allows ε_t^w not to interfere with the micro-foundations of eq. (1.3). Readers who do not like this story may simply assume that ε_t^w is equal to 0 everywhere. This shock does not play any role in the main results of this paper. It is simply a convenient way to introduce a random profitability shock in the model.

with a lag of one period⁴. However all agents observe the current value of nominal financial asset prices, exchange rates and interest rates. Contrary to aggregate price and output data, financial variables are immediately observable on the markets in which they are quoted. We also assume that the aggregate demand disturbance is first order autocorrelated, $\varepsilon_t^d = \delta\varepsilon_{t-1}^d + v_t$ where v_t is white noise, while the supply shock is a random walk, $\varepsilon_t^s = \varepsilon_{t-1}^s + \nu_t$ where ν_t is white noise. The other shocks, ε_t^l , ε_t^x and ε_t^w , are supposed to be white noise. Note that if ε_t^l was a bubble it should be such that $\varepsilon_t^l = \rho E_t \varepsilon_{t+1}^l$ which is not compatible with the white noise hypothesis.

Equations (2.1), (2.2), (2.3) and (2.4) combine to yield

$$p_t = E_{t-1}p_t - \frac{1}{\theta}(\alpha - \beta(R_t - E_t\pi_{t+1}) + \gamma l_t + \varepsilon_t^d - \mu(e_t + p_t - p_t^*) - \varepsilon_t^s - \bar{y}) \quad (2.11)$$

which using (2.7) implies that

$$\begin{aligned} p_t(1 + \frac{\mu}{\theta}) &= E_{t-1}p_t + \frac{\alpha}{\theta} - \frac{\bar{y}}{\theta} + \frac{\beta}{\theta}E_t\pi_{t+1} + \frac{\gamma}{\theta}L_t - \frac{\gamma}{\theta}E_tp_t - \frac{\mu}{\theta}e_t + \frac{\mu}{\theta}p_t^* - \frac{\beta}{\theta}R_t \\ &\quad + \frac{1}{\theta}(\varepsilon_t^d - \varepsilon_t^s) \end{aligned} \quad (2.12)$$

This is the rule that prices p_t must follow to clear the goods market. Since the current price level p_t is unobservable, people may only rely on a guess about this price level using available information. This guess is the price perception E_tp_t . Taking the expectation of both sides of equation 2.12, we see that E_tp_t must follow

$$\begin{aligned} E_tp_t(1 + \frac{\mu}{\theta}) &= E_{t-1}p_t + \frac{\alpha}{\theta} - \frac{\bar{y}}{\theta} + \frac{\beta}{\theta}E_t\pi_{t+1} + \frac{\gamma}{\theta}E_tL_t - \frac{\gamma}{\theta}E_tp_t - \frac{\mu}{\theta}E_te_t + \frac{\mu}{\theta}p_t^* \\ &\quad - \frac{\beta}{\theta}E_tR_t + \frac{1}{\theta}E_t(\varepsilon_t^d - \varepsilon_t^s) \end{aligned} \quad (2.13)$$

Let η_t be the price perception error $p_t - E_tp_t$. Since $E_tL_t = L_t$, $E_te_t = e_t$ and $E_tR_t = R_t$, it is easy to derive the value of η_t , subtracting eq. 2.13 from eq.2.12:

$$p_t - E_tp_t = \frac{1}{\theta + \mu} \{(\varepsilon_t^d - \varepsilon_t^s) - E_t(\varepsilon_t^d - \varepsilon_t^s)\} = \eta_t \quad (2.14)$$

The concepts of price perception and price perception error will prove to be particularly useful when deriving the optimal monetary policy.

⁴An individual agent may contemporaneously observe the output and price levels of the particular market on which he is acting, but not the aggregate values of all the markets.

3. Optimal monetary policy under output gap stabilisation and inflation rate targeting

In this section we derive the optimal monetary policy for a particular form of authority's objective, following Broadbent and Barro (1997). The loss function that the Central Bank minimizes is a weighted sum of squared deviations from an output target of $\bar{y} + \varepsilon_t^s$ and an inflation target of $\bar{\pi}$. A strictly positive value of χ may involve the governments's optimal rate of taxation of cash balances, since the government obtains revenue from printing money (see for example Barro and Gordon (1983), with however a quite different output target). Formally, the policy maker's objective is

$$\min_{R_t} E_t((y_t - \bar{y} - \varepsilon_t^s)^2 + \chi(\pi_t - \bar{\pi})^2) \quad (3.1)$$

There is an arbitrage between minimizing the output gap and stabilizing the inflation rate. The coefficient χ is the relative weight the authorities attach to attaining their inflation target. This loss function reduces to

$$\min E_t \left(\theta^2 (p_t - E_{t-1} p_t)^2 + \chi (p_t - p_{t-1} - \bar{\pi})^2 \right)$$

using eqs. (2.2) and (2.3). Indeed the output gap depends on the price forecast error. In this model the Central Bank sets the interest rate level in order to control the price level⁵. There is an arbitrage between hitting the inflation target and holding down forecast errors in the aggregate price level. However, the monetary authorities cannot control the aggregate price level perfectly because they are not contemporaneously observable. This feature inevitably introduces a degree of error in the control of prices, equal to η_t . The Central Bank may only control the perceived price $E_t p_t$. The monetary authorities select an optimal expected equilibrium point on the aggregate supply curve, corresponding to some particular value of $E_t p_t$, where expected output is equal to $\bar{y} + \theta(E_t p_t - E_{t-1} p_{t-1}) + \varepsilon_{t-1}^s$. The Central Bank then controls the nominal interest rate to shift aggregate demand and thereby hits the desired expected equilibrium point. Therefore we may first think of the Central Bank as picking an optimal value for perceived prices $E_t p_t$ although we can ultimately express the results in terms of settings for the nominal interest rate level R_t .

The Central Bank minimizes its objective in each period, taking as given the

⁵In this paper monetary policy is entirely defined as a feedback rule and does not contain an unsystematic component or monetary shock. For a discussion of the issues implied by such shocks, see Christiano and al. (1998).

prior expectations $E_{t-1}p_t$ since we assume that policy is conducted without a pre-commitment ability. But expectations are rational, so that private agents know that monetary authorities are in this position. Private agents formulate their expectations by effectively solving the problem that the Central Bank has to solve. Monetary policy is thus the outcome of a non-cooperative game between the Central Bank and private agents. However, since the objective function is such that there is no reason to behave in a time-inconsistent manner, the rules-based and discretionary solutions would coincide in this case.

In the next developments we use a novel methodology, introduced by Broadbent and Barro (1997), which avoids some of the cumbersome calculation associated with solving rational expectations models with an explicit goal for the monetary authority. Conventional techniques generally require solving the whole model before substituting this solution into the objective function and deriving the optimal rule. Here we derive the optimal monetary policy rule before solving the model.

It is thus convenient to rewrite the Central Bank's objective in terms of perceived prices⁶:

$$\min_{E_t p_t} E_t \left(\theta^2 (E_t p_t + \eta_t - E_{t-1} p_t)^2 + \chi (E_t p_t + \eta_t - p_{t-1} - \bar{\pi})^2 \right) \quad (3.2)$$

The first-order condition characterizing the optimum implies

$$(\theta^2 + \chi) E_t p_t = \theta^2 E_{t-1} p_t + \chi (p_{t-1} + \bar{\pi}) \quad (3.3)$$

and taking expectations of this result as of date $t-1$ leads to

$$(\theta^2 + \chi) E_{t-1} p_t = \theta^2 E_{t-1} p_t + \chi (E_{t-1} p_{t-1} + \bar{\pi}) \quad (3.4)$$

⁶It may be interesting to note that the objective function of the Central Bank could have been initially written as an intertemporal choice problem:

$$\min_{E_t \pi_t, E_{t+1} p_{t+1}, \dots} \left(\sum_{i=0}^{\infty} \left(\frac{E_t (\theta^2 (E_{t+i} \pi_{t+i} + \eta_{t+i} - E_{t+i-1} \pi_{t+i})^2 + \chi (E_{t+i} \pi_{t+i} - \bar{\pi})^2)}{(1+\delta)^i} \right) \right)$$

where δ is a constant discount rate. However, to derive the solution for $E_t \pi_t$, this intertemporal objective reduces to a one-period choice problem:

$$\min_{E_t \pi_t} E_t \left(\theta^2 (E_t \pi_t + \eta_t - E_{t-1} \pi_t)^2 + \chi (E_t \pi_t + \eta_t - \bar{\pi})^2 \right)$$

which is equivalent to eq. (3.2). Two reasons explain why an intertemporal problem reduces here to a static one. The first one is that the structure of the model is such that future expectations $E_{t+i-1} \pi_{t+i}$ are independent of $E_t \pi_t$. The second reason is that the current choice of the value of $E_t \pi_t$ has no impact on future output gaps and implies no direct constraints on future choices $E_{t+i} \pi_{t+i}$. There is thus no channel for $E_t \pi_t$ to affect future costs in the intertemporal loss function.

which reduces to

$$E_{t-1}p_t = E_{t-1}p_{t-1} + \bar{\pi} = p_{t-1} - \eta_{t-1} + \bar{\pi} \quad (3.5)$$

Eq. (3.3) then becomes

$$(\theta^2 + \chi)E_t p_t = \theta^2(E_{t-1}p_{t-1} + \bar{\pi}) + \chi(p_{t-1} + \bar{\pi}) \quad (3.6)$$

which implies

$$E_t p_t = \frac{\theta^2 E_{t-1} p_{t-1} + \chi p_{t-1}}{\theta^2 + \chi} + \bar{\pi} = \frac{(1 + \chi')p_{t-1} - \eta_{t-1}}{1 + \chi'} + \bar{\pi} = p_{t-1} + \bar{\pi} - \frac{\eta_{t-1}}{1 + \chi'} \quad (3.7)$$

where $\chi' = \frac{\chi}{\theta^2}$.

The output gap then becomes from eqs. 2.3, 2.14, 3.5 and 3.7 :

$$y_t - \bar{y} - \varepsilon_t^s = \theta(E_t p_t + \eta_t - E_{t-1} p_t) = \theta \eta_t + \frac{\theta \chi'}{1 + \chi'} \eta_{t-1} \quad (3.8)$$

Equation (3.7) summarizes the optimal monetary policy which can also be described in terms of perceived inflation:

$$E_t \pi_t = E_t(p_t - p_{t-1}) = E_t p_t - p_{t-1} = \frac{\theta^2(E_{t-1} p_{t-1} - p_{t-1})}{\theta^2 + \chi} + \bar{\pi} = \bar{\pi} - \frac{\eta_{t-1}}{1 + \chi'} \quad (3.9)$$

The monetary authorities enforce a perceived inflation rate which is equal to the target adjusted for the previous perception error. The unobservability of the price level for one period indeed leads to the presence of the price perception error on date t-1 in the optimal price perception on date t. This feature, which might seem confusing at first sight, is a mechanical implication of the model and is discussed by Broadbent and Barro (1997). It is easy to derive some useful additional results:

$$E_t p_{t+1} = E_t(E_{t+1} p_{t+1}) = E_t(p_t - \frac{\eta_t}{1 + \chi'} + \bar{\pi}) = p_{t-1} + 2\bar{\pi} - \frac{\eta_{t-1}}{1 + \chi'} \quad (3.10)$$

and

$$E_t \pi_{t+1} = E_t p_{t+1} - E_t p_t = \bar{\pi} = E_{t-1} \pi_t \quad (3.11)$$

The task of the Central Bank is to adjust the nominal interest rate in response to the various shocks affecting the economy, to make sure that $E_t p_t$ follows (3.7). Therefore shocks such as ε^x and ε^l directly influence the setting of nominal interest rates. However, there is a 'feedback effect' of nominal interest rates on financial asset prices L and e . By setting the nominal interest rates, the Central Bank also affects the levels of stock prices and the exchange rate. Nominal interest rates affect aggregate demand directly (the intensity of this direct effect is measured by β), and indirectly through the channel of stock prices and exchange rates. Financial variables R , L and e are determined simultaneously. We address these issues in the next section.

4. Solution of the model

4.1. A monetary conditions indicator

Substituting eqs. (3.5), (3.7) and (3.11) into (2.13), and rearranging, we obtain

$$\begin{aligned}
 R_t = & (-\gamma - \mu) \frac{1}{\beta} (p_{t-1} + \bar{\pi}) - \left(\frac{\theta \chi' - \mu - \gamma}{\beta(1 + \chi')} \right) \eta_{t-1} + \bar{\pi} + \frac{(\alpha - \bar{y} + \mu E_t p_t^*)}{\beta} + \frac{\gamma}{\beta} L_t \\
 & - \frac{\mu}{\beta} e_t + \frac{1}{\beta} E_t (\varepsilon_t^d - \varepsilon_t^s)
 \end{aligned} \tag{4.1}$$

This equation may be interpreted as a rule that the nominal interest rate level must follow, but it is not a reduced form solution since L_t and e_t are endogenous. It may be pointed out that, in this relation, interest rates are a positive function of equity prices but a negative one of exchange rates. This observation must be interpreted with care since all variables are endogenous. For example, a positive exogenous shock on equity prices ($\epsilon_t^l < 0$) boosts aggregate demand and the Central Bank reacts by increasing the intervention interest rate. But this is not the whole story: higher interest rates have a negative effect on equity prices (which moderates the initial hike) and a negative effect on exchange rates. Similarly, a positive exogenous shock on the exchange rate ($\epsilon_t^x < 0$) depresses aggregate demand and induces a downward adjustment of the interest rate by monetary authorities, which involves a lot of additional variations of financial asset prices.

Using $E_t(\varepsilon_t^d - \varepsilon_t^s) = \delta \varepsilon_{t-1}^d - \varepsilon_{t-1}^s$, eq. (4.1) implies that the Central Bank controls the nominal interest rate in such a way that a weighted average of interest rates, stock prices and exchange rates responds adequately to date t-1

demand and supply shocks. This weighted average represents what is called a monetary condition indicator (*MCI*) by Central Bank policy makers :

$$MCI_t = \beta R_t - \gamma L_t + \mu e_t = c_t + \delta \varepsilon_{t-1}^d - \varepsilon_{t-1}^s \quad (4.2)$$

where $c_t = -(\gamma + \mu)(p_{t-1} + \bar{\pi}) + \beta \bar{\pi} + (\alpha - \bar{y} + \mu E_t p_t^*) - (\frac{\theta \chi' - \mu - \gamma}{(1 + \chi')}) \eta_{t-1}$.

The weights of the components in the MCI depend on their respective effect on aggregate demand. Absent c_t , eq. (4.2) is a general form in which some particular cases proposed in the recent literature are nested. If $\gamma = 0$, eq. (4.2) reduces to the case studied by Gerlach and Smets (1996) while if $\mu = 0$, eq. (4.2) corresponds to the case analysed by Smets (1997). Notice that in this framework contemporaneous shocks to supply and demand are unobservable, the MCI depends upon past supply and demand shocks.

To interpret eq. (4.2) correctly, it must be remembered that R and L are endogenous. To set MCI_t at its desired value, the Central Bank does not move R_t conditional to exogenous values of equity prices and exchange rates. To achieve the equality described by (4.2), some fine tuning of interest rates is necessary, taking account of the fact that equity prices and exchange rates react to movements in interest rates.

4.2. Explicit solutions for interest rates, exchange rates and stock prices

Using equation (2.7), the arbitrage condition eq. (2.5) may be rewritten in nominal terms:

$$L_t = E_t p_t - R_t + E_t \pi_{t+1} - \rho E_t (L_{t+1} - E_{t+1} p_{t+1}) + (1 - \rho) E_t d_{t+1} - \varepsilon_t^l \quad (4.3)$$

Substituting eqs. (2.6) and (2.8) into this equation yields:

$$\begin{aligned} L_t = & E_t p_t - R_t + E_t \pi_{t+1} - \rho E_t L_{t+1} - \rho E_t (E_{t+1} p_{t+1}) + (1 - \rho) E_t (y_t - \tau \\ & - \lambda(l w_{t-1} + E_{t-1} \pi_t + \varepsilon_t^w - p_t)) - \varepsilon_t^l \end{aligned} \quad (4.4)$$

Substituting eqs. (3.8) and (4.1) into eq. (4.3), using eqs. (3.7) and (3.11), we obtain:

$$\begin{aligned}
L_t = & \left(\frac{(2(1-\rho)-1)\beta + \gamma + \mu}{\beta + \gamma} \right) \bar{\pi} + \left(\frac{(1-\rho)(\bar{y} - \tau)\beta - (\alpha - \bar{y} + \mu E_t p_t^*)}{\beta + \gamma} \right) \\
& + \left(\frac{\mu}{\beta + \gamma} \right) e_t + \left(\frac{\beta(1-\rho) + \gamma + \mu}{\beta + \gamma} \right) p_{t-1} - \frac{\beta(1-\rho)\lambda(lw_{t-1} - p_{t-1})}{\beta + \gamma} \\
& + \frac{\beta\rho}{\beta + \gamma} E_t L_{t+1} + \left(\frac{\beta \left[(1-\rho)E_t \varepsilon_t^s - (1-\rho)\lambda E_t \varepsilon_t^w - \varepsilon_t^l \right] - E_t(\varepsilon_t^d - \varepsilon_t^s)}{\beta + \gamma} \right) \\
& - \frac{\beta(1-\rho)(1 + \lambda - \theta\chi')}{(\beta + \gamma)(1 + \chi')} \eta_{t-1} \tag{4.5}
\end{aligned}$$

Similarly, substituting eq. (4.1) into the uncovered interest rate parity condition eq. (2.10) yields

$$\begin{aligned}
e_t = & R_t - R_t^* + E_t e_{t+1} - \varepsilon_t^x \\
= & (-\gamma - \mu) \frac{1}{\beta} (p_{t-1} + \bar{\pi}) + \bar{\pi} + \frac{(\alpha - \bar{y} + \mu E_t p_t^*)}{\beta} + \frac{\gamma}{\beta} L_t - \frac{\mu}{\beta} e_t + \frac{1}{\beta} (\delta \varepsilon_{t-1}^d - \varepsilon_{t-1}^s) \\
& - R_t^* + E_t e_{t+1} - \varepsilon_t^x - \left(\frac{\theta\chi' - \mu - \gamma}{\beta(1 + \chi')} \right) \eta_{t-1} \tag{4.6}
\end{aligned}$$

Collecting the terms in e_t , eq. (4.6) implies that :

$$\begin{aligned}
e_t = & \left(\frac{(-\gamma - \mu)}{\beta + \mu} \right) p_{t-1} + \left(\frac{\beta - \gamma - \mu}{\beta + \mu} \right) \bar{\pi} + \left(\frac{\alpha - \bar{y} + \mu E_t p_t^*}{\beta + \mu} \right) + \left(\frac{\gamma}{\beta + \mu} \right) L_t \\
& - \left(\frac{\beta}{\beta + \mu} \right) R_t^* + \left(\frac{\beta}{\beta + \mu} \right) E_t e_{t+1} + \left(\frac{1}{\beta + \mu} \right) E_t (\varepsilon_t^d - \varepsilon_t^s) - \left(\frac{\beta}{\beta + \mu} \right) \varepsilon_t^x \\
& - \left(\frac{\theta\chi' - \mu - \gamma}{(\beta + \mu)(1 + \chi')} \right) \eta_{t-1} \tag{4.7}
\end{aligned}$$

Eqs. (4.5) and (4.7) form a system of two linear equations in L_t and e_t conditioned on expectations $E_t e_{t+1}$ and $E_t L_{t+1}$. Taking these expectations as given, the solution of this system is:

$$L_t = \left(\frac{-(1-\rho)\lambda(lw_{t-1} - p_{t-1})(\beta + \mu)}{\beta + \gamma + \mu} \right)$$

$$\begin{aligned}
& + \left(\frac{(\beta + \mu)((1 - \rho)(\bar{y} - \tau)) - (\alpha - \bar{y} + \mu p_t^*)}{\beta + \gamma + \mu} \right) - \left(\frac{\mu}{\beta + \gamma + \mu} \right) R_t^* \\
& + \left(\frac{(\beta + \mu)(1 - \rho) + (\gamma + \mu)}{\beta + \gamma + \mu} \right) p_{t-1} + \left(\frac{2(\beta + \mu)(1 - \rho) - (\beta - \gamma - \mu)}{\beta + \gamma + \mu} \right) \bar{\pi} \\
& + \left(\frac{(\beta + \mu)\rho}{\beta + \gamma + \mu} \right) E_t L_{t+1} + \left(\frac{\mu}{\beta + \gamma + \mu} \right) E_t e_{t+1} - \left(\frac{\mu}{\beta + \gamma + \mu} \right) \varepsilon_t^x \\
& - \left(\frac{1}{\beta + \gamma + \mu} \right) E_t (\varepsilon_t^d - \varepsilon_t^s) + \left(\frac{(\beta + \mu)(1 - \rho)}{\beta + \gamma + \mu} \right) E_t \varepsilon_t^s \\
& - \left(\frac{(\beta + \mu)(1 - \rho)\lambda}{\beta + \gamma + \mu} \right) E_t \varepsilon_t^w - \left(\frac{(\beta + \mu)}{\beta + \gamma + \mu} \right) \varepsilon_t^l \\
& + \left(\frac{\mu(\mu + \gamma - \theta\beta\chi') - (1 - \rho)(1 + \lambda - \theta\chi')\beta(\beta + \mu)}{\beta(\beta + \gamma + \mu)(1 + \chi')} \right) \eta_{t-1} \tag{4.8}
\end{aligned}$$

$$\begin{aligned}
e_t = & \left(\frac{2\gamma(1 - \rho) + \beta - \gamma - \mu}{\beta + \gamma + \mu} \right) \bar{\pi} - \left(\frac{(\gamma\rho + \mu)}{\beta + \gamma + \mu} \right) p_{t-1} - \left(\frac{\beta + \gamma}{\beta + \gamma + \mu} \right) R_t^* + \\
& \left(\frac{\alpha - \bar{y} + \mu E_t p_t^* + \gamma(1 - \rho)(\bar{y} - \tau)}{\beta + \gamma + \mu} \right) - \left(\frac{\gamma(1 - \rho)\lambda(lw_{t-1} - p_{t-1})}{\beta + \gamma + \mu} \right) \\
& + \left(\frac{\beta + \gamma}{\beta + \gamma + \mu} \right) E_t e_{t+1} + \left(\frac{\gamma\rho}{\beta + \gamma + \mu} \right) E_t L_{t+1} \\
& + \frac{E_t(\varepsilon_t^d - \varepsilon_t^s) - (\gamma + \beta)\varepsilon_t^x + \gamma([1 - \rho] E_t \varepsilon_t^s - [1 - \rho] \lambda \varepsilon_t^w - \varepsilon_t^l)}{\beta + \gamma + \mu} \\
& + \frac{(\beta + \gamma)(\mu + \gamma - \theta\beta\chi') - \gamma(1 - \rho)(1 + \lambda - \theta\chi')\beta}{(\beta + \gamma + \mu)\beta(1 + \chi')} \eta_{t-1} \tag{4.9}
\end{aligned}$$

Eqs. (4.8) and (4.9) form a system of two linear first-order difference equations in the nominal stock price L_t and the nominal exchange rate e_t . This system is solved in Appendix 3, where the appropriate dynamic stability conditions are derived, and yields the following solutions :

$$\begin{aligned}
L_t = & \bar{\pi} + p_{t-1} - \frac{1}{1 - \rho} R^* + \frac{1}{1 - \rho} \bar{\pi}^* + \bar{y} - \tau - \lambda(lw_{t-1} - p_{t-1}) - \frac{\mu}{\beta + \gamma + \mu} \varepsilon_t^x \\
& - \frac{(\beta + \mu)(1 - \rho)\lambda}{(\beta + \gamma + \mu)} \varepsilon_t^w - \frac{(\beta + \mu)}{(\beta + \gamma + \mu)} \varepsilon_t^l + \varepsilon_{t-1}^s
\end{aligned}$$

$$+\frac{\delta(\delta-1)}{\beta(1-\rho\delta)(1-\delta)+\gamma(1-\delta)+\mu(1-\rho\delta)}\varepsilon_{t-1}^d \quad (4.10)$$

$$\begin{aligned} e_t = & -\bar{\pi} - p_{t-1} - \frac{\beta(1-\rho)+\gamma}{\mu(1-\rho)}(R^* - \bar{\pi}^*) + p_{t-1}^* + \bar{\pi}^* \\ & + \frac{\alpha - \bar{y} + \mu E_t p_t^* + \gamma(\bar{y} - \tau) - \gamma\lambda(lw_{t-1} - p_{t-1})}{\mu} - \frac{\gamma(1-\rho)\lambda}{\beta + \gamma + \mu}\varepsilon_t^w \\ & - \frac{\gamma}{\beta + \gamma + \mu}\varepsilon_t^l - \frac{\gamma + \beta}{\beta + \gamma + \mu}\varepsilon_t^x - \frac{(1-\gamma)}{\mu}\varepsilon_{t-1}^s \\ & + \frac{\delta(1-\rho\delta)}{\beta(1-\rho\delta)(1-\delta)+\gamma(1-\delta)+\mu(1-\rho\delta)}\varepsilon_{t-1}^d \end{aligned} \quad (4.11)$$

In the solutions above we have neglected the terms in η_{t-1} which do not deserve any interest (the value of η_{t-1} is arbitrary). We also assumed that the foreign interest rate is constant ($R_t^* = R^*, \forall t$), and that expectations about foreign prices are formulated according to the simple rule : $E_t p_{t+i}^* = p_{t-1}^* + (i+1)\bar{\pi}^*$, where $\bar{\pi}^*$ is the constant foreign inflation rate. Substituting these solutions in eq. (4.2) where $E_t p_t^* = p_{t-1}^* + \bar{\pi}^*$, we obtain the explicit solution for the interest rate R_t :

$$\begin{aligned} R_t = & R^* + \bar{\pi} - \bar{\pi}^* + \frac{\mu}{\beta + \gamma + \mu}\varepsilon_t^x - \frac{\gamma(\beta - \mu\rho)(1-\rho)\lambda}{\beta(\beta + \gamma + \mu)}\varepsilon_t^w - \frac{\gamma}{\beta + \gamma + \mu}\varepsilon_t^l \\ & + \frac{\delta(1-\rho\delta)(1-\delta)}{\beta(1-\rho\delta)(1-\delta)+\gamma(1-\delta)+\mu(1-\rho\delta)}\varepsilon_{t-1}^d \end{aligned} \quad (4.12)$$

Note that the variance of equilibrium interest rates is given by :

$$\begin{aligned} \sigma_R^2 = & \left(\frac{\mu}{\beta + \gamma + \mu} \right)^2 \sigma_{\varepsilon^x}^2 + \left(\frac{\gamma(\beta - \mu\rho)(1-\rho)\lambda}{\beta(\beta + \gamma + \mu)} \right)^2 \sigma_{\varepsilon^w}^2 + \left(\frac{\gamma}{\beta + \gamma + \mu} \right)^2 \sigma_{\varepsilon^l}^2 \\ & + \left(\frac{\delta(1-\rho\delta)(1-\delta)}{\beta(1-\rho\delta)(1-\delta)+\gamma(1-\delta)+\mu(1-\rho\delta)} \right)^2 \sigma_{\varepsilon^d}^2 \end{aligned} \quad (4.13)$$

under the assumption that ε_{t-1}^d , ε_t^x , ε_t^l and ε_t^w are mutually independent. The solutions of the model may thus be restated in terms of volatilities.

4.3. Discussion

In this subsection we comment on the above solutions. In particular we discuss the effect of various random shocks and exogenous variables on equity prices, exchange rates and interest rates.

From the discussion which follows, some important lessons may be drawn.

- When monetary policy takes account of the role played by financial prices in the monetary transmission mechanism the optimal monetary policy reaction to a demand shock is inferior to what it would have been otherwise. This reduction of the magnitude of the interest rates reaction increases with the impact of financial prices on aggregate demand.

- This feature of the solutions has interesting implications since there are good reasons to assume that the impacts of financial prices on aggregate demand change over time. For example it may be suspected that the sensibility of consumer demand with respect to the value of equities has recently increased as the result of increasing stock holding by households. Therefore γ should have increased. An important consequence of this structural change is that the optimal monetary policy reaction to demand shocks should have decreased. This argument would imply a reduction of the volatility of intervention interest rates for a given level of the variance of aggregate demand.

- In an inflation targeting framework, monetary policy must react to unanticipated changes in asset prices only to the extent that they affect the output gap and thus the inflation outlook. To correctly assess the optimal response of interest rates to exogenous changes of the stochastic risk premia on equity prices and exchange rates, it is necessary to include both kinds of financial prices in the model because of their interaction.

4.3.1. Effects of supply and demand shocks

When setting interest rates, the Central Bank targets perceived prices $E_t p_t$. Equity prices and exchange rates also depend on expectations conditioned on information available on period t (see eqs. (2.5) and (2.10)). Because contemporary supply and demand shocks ε_t^s and ε_t^d are not in the date t information set, they do not affect expectations made on date t about future variables. This is the reason why they do not influence R_t , L_t , and e_t .

However past supply and demand shocks may have some effect on L_t and e_t for several reasons.

Effects of past demand shocks Perceived prices, which must be equalized to their optimal level given by eq. 2.7, depend on expected demand shocks to the output gap (see eq.(2.13)). Expected demand shocks in turn are a function of past supply and demand shocks. Therefore the interest rate, which is controlled to hit the desired level of perceived prices, is affected positively by past demand shocks. Being a function of interest rates (see eqs. (2.5) and (2.7)), nominal asset prices also depend on ε_{t-1}^d at equilibrium.

Equity prices decrease with a positive past demand shock because interest rates increase, while expected future dividends are unaffected since expected future output remains constant. Nominal exchange rates increase with a positive past demand shock because interest rates increase.

For given volatilities of the other stochastic shocks, the effect of the volatility of demand shocks on the volatility of interest rates is a negative function of μ and γ . Indeed,

$$\frac{\partial \sigma_R^2}{\partial \sigma_{\varepsilon^d}^2} = \left(\frac{\partial R_t}{\partial \varepsilon_{t-1}^d} \right)^2 = \left(\frac{\delta(1 - \rho\delta)(1 - \delta)}{\beta(1 - \rho\delta)(1 - \delta) + \gamma(1 - \delta) + \mu(1 - \rho\delta)} \right)^2$$

which clearly shows that, for a given variance of demand shocks, the volatility of interest rates is smoothed by the impacts of equity prices and exchange rates on aggregate demand. Figure 1 in appendix 4 (see document .gif) illustrates the dependence of $\frac{\partial R_t}{\partial \varepsilon_{t-1}^d}$ on μ and γ assuming that $\rho = .96$, $\delta = .50$ and $\beta = .10$ ⁷.

It is important to point out that the way $\frac{\partial R_t}{\partial \varepsilon_{t-1}^d}$ depends on μ is affected by the value of γ : for a given value of μ , $\frac{\partial R_t}{\partial \varepsilon_{t-1}^d}$ is a negative function of γ . Similarly for a given value of γ , the derivative $\frac{\partial R_t}{\partial \varepsilon_{t-1}^d}$ is a negative function of μ .

Effects of past supply shocks We have assumed that supply shocks are permanent. This hypothesis implies that past supply shock ε_{t-1}^s have no effects on the equilibrium interest rate. This absence of any influence of ε_{t-1}^s on R_t is indeed necessary to allow the uncovered interest rates parity condition (2.10) to hold⁸. Since supply shocks are permanent, the effect of ε_{t-1}^s on e_t is the same as

⁷In US data over the period 1926 to 1994, ρ is about .96 in annual data (see Campbell and al., 1997 or Cogley, 1999). This is a prudent choice since this estimate would be higher if the recent run-up in stock prices was included in the sample. The value of δ is arbitrarily chosen as the mean of its acceptable values. It is very hard to estimate β due to the simultaneity of real demand and interest rates (see for example Barro and Broadbent, 1997). Therefore a small value like $\beta = .10$ is a conservative choice.

⁸For the same reason ε_{t-1}^d has no effect on R_t if this shock is permanent ($\delta = 1$).

the effect of ε_{t-1}^s on $E_t e_{t+1}$ so that the right-side of (2.10) remains unchanged. Therefore R_t must also remain unchanged on the left-side of (2.10). This result in turn explains why past supply shocks ε_{t-1}^s have a unit coefficient in the equilibrium equity price equation. This coefficient reflects the fact that, at a stationary equilibrium and in the absence of any reaction of R_t to ε_{t-1}^s , expected real asset prices have a unit elasticity with respect to expected real dividend (an implication of eq. (2.5)). Real dividends are proportional to real output and, from eq. (2.3), expected real output in turn depends on ε_{t-1}^s with a unit coefficient. In response to a positive value of ε_{t-1}^s aggregate demand must increase to stabilize the output gap. Since interest rates do not react, this adjustment operates through an increase of equity prices and a decrease of exchange rates. The necessary downwards adjustment of exchange rates therefore depends on the relative impact of both kinds of assets on aggregate demand. This argument explains that the magnitude of the negative effect of ε_{t-1}^s on e_t decreases when γ increases. If $\gamma = 1$, a variation of e_t is unnecessary since $\frac{\partial L_t}{\partial \varepsilon_{t-1}^s} = 1$.

4.3.2. Effects of financial shocks

It may be observed that ε_t^l has a negative effect on equity prices, interest rates and exchange rates. A random increase of the risk premium on the stock market ($\varepsilon_t^l > 0$) decreases equity prices (see eq. (2.5)) and depresses aggregate demand (see eq. (2.1)). In response the Central Bank adjusts interest rates downwards, which implies a decline of exchange rates (see eq. (2.10)). These reactions describe the initial effects of the shock. The initial effects are the most important to determine the final outcome, but the full path to the equilibrium solution involves additional interactions between the variables. The decline of exchange rates lowers the decrease of aggregate demand and makes unnecessary a continuation of the downward movement of interest rates. Moreover the decline of interest rates has a positive effect on equity prices (an effect which does not fully offset the initial depression of equity prices), which feeds into aggregate demand (already directly supported by declining interest rates) and stabilizes the system.

The dependence of the monetary policy reaction to ε_t^l on μ and γ is illustrated by figure 2 (see appendix 4 in document .gif), using the same value as above for β .

The negative effect of ε_t^l on equity prices increases with the magnitude of β and μ . A higher value of μ implies that a smaller depreciation of the exchange

rate is necessary to make aggregate demand go up again to its equilibrium value. A smaller depreciation of the exchange rate necessitates a smaller decrease of interest rates. A higher value of β implies that a smaller fall of interest rates is required to push up aggregate demand to its equilibrium value. In both cases a smaller fall of interest rates implies that stocks go less up again after the initial fall. Therefore the initial fall is less offset by the upward movement caused by the fall of interest rates. The negative effect of ε_t^l on the exchange rate increases with the magnitude of γ .

The speculative shock on currency markets, ε_t^x , has a negative effect on exchange rates and equity prices, but a positive effect on interest rates. A speculation attack against the domestic currency ($\varepsilon_t^x > 0$) depreciates the exchange rate and boosts aggregate demand. In response to this variation of the output gap, the Central Bank increases the interest rate. Equity prices then decline. Declining equity prices and increasing interest rates cause aggregate demand to fall back after its initial surge. Increasing interest rates also cause the exchange rate to go up again, without fully offsetting its initial fall. The dependence of the monetary policy reaction to ε_t^x on μ and γ is illustrated by figure 3 (see appendix 4 in document .gif), using the same value as above for β .

The negative effect of ε_t^x on the equilibrium exchange rate increases with the magnitude of β and γ . A higher value of β implies that a smaller increase of interest rates is sufficient to make aggregate demand fall to the equilibrium level of output. Similarly, a higher value of γ implies that a smaller fall of equity prices, and therefore a smaller increase of interest rates, is sufficient to make aggregate demand fall back to its equilibrium level. A smaller rise of interest rates in turn implies that the exchange rate does less go up again and does less offset its initial fall.

The negative effect of ε_t^x on equity prices increases with the magnitude of μ . A bigger value of μ indeed implies a bigger increase of aggregate demand in response to the depreciation of the exchange rate. This bigger hike of aggregate demand necessitates a bigger increase of interest rates to restore equilibrium, and causes therefore a bigger fall of equity prices.

4.3.3. Effects of profitability

An increase of the equilibrium value of real profits $\bar{y} - \tau - \lambda(lw_{t-1} - p_{t-1})$ increases equity prices. The partial derivative of equity prices to real dividends is indeed equal to one, which reflects the long term unit elasticity of real equity prices

to real dividends (see eq. (2.5)). Real profits also have a positive effect on the nominal exchange rate, increasing with γ and decreasing with μ . A higher value of γ implies that a given increase of real dividends (a smaller value of τ caused by an increase of productivity, or a fall of real wages), implying a proportional increase of real stock prices, has a bigger effect on aggregate demand and requires a bigger increase of interest rates to push down aggregate demand. A higher value of μ implies that the induced appreciation of the exchange rate leads to a sharper downward reaction of aggregate demand, implying that a smaller appreciation is necessary to push down aggregate demand. The net effect of an increase of profitability on interest rates is null.

4.3.4. Effects of foreign interest rates and inflation

In the equilibrium interest rate solution, the constant term $R^* + \bar{\pi} - \bar{\pi}^*$ is a logical outcome. Absent stochastic terms, domestic interest rates are equal to foreign interest rates plus the long term expected inflation differential. Equity prices decrease when the foreign interest rate goes up. A hike of foreign interest rates initially implies a depreciation of the exchange rates which boosts aggregate demand. The Central Bank reacts and raises interest rates. Equity prices then fall. In the exchange rate solution, the term $-\bar{\pi} - p_{t-1} + p_{t-1}^* + \bar{\pi}^*$ reflects a kind of purchasing power parity condition. Of course, a hike of real foreign interest rates ($R^* - \bar{\pi}^*$) depreciates the domestic exchange rate.

5. Interaction of monetary and fiscal policies under a public debt solvency constraint

In this section we introduce a fiscal policy component g_t in aggregate demand, so that equation 1.1 becomes

$$y_t^d = \alpha + g_t - \beta(R_t - E_t\pi_{t+1}) + \gamma l_t - \mu(e_t + p_t - p_t^*) + \varepsilon_t^d \quad (5.1)$$

The new variable g_t measures the real primary public deficit per unit of natural output. The primary deficit is the difference between public expenses without interest payments and public receipts. The real primary deficit is divided by the natural output level to obtain g_t . The fiscal policy variable g_t must respect a public debt solvency constraint, inspired by Krichel, Livine and Pearlman (1996), and Van Aarle, Bovenberg and Raith (1995). This very simple constraint requires that the real public debt level remains constant (see also

Artus (1998)). This particular constraint is the right one in a model where expected output is equal to a constant natural level. In this context indeed, such a constraint is equivalent to requiring that the ratio of real debt on output be constant. The solvency constraint establishes a strong link between fiscal and monetary policies, since the latter determines the level of inflation and therefore the level of seignorage. Since some form of real debt stabilization requirement is binding in most industrial countries, with the example of the European Stability Pact, it is important to address the question of how such a constraint modifies the optimal reaction of monetary policy to asset prices fluctuations.

After linearisation (see Appendix 4), this constraint requires that :

$$\pi_t - R_t = g_t \frac{\bar{Y}}{X_{t-1}} \quad (5.2)$$

where \bar{Y} is the natural level of production ($\bar{y} = \ln \bar{Y}$), and X is the real public debt. For a given interest rate, an increase of the real primary deficit, augmenting the real debt, has to be compensated by an increase of inflation (seignorage) to reduce the real public debt⁹.

We have solved the new model obtained by replacing equation (2.1) with equation (5.1) and adding equation (5.2). Our main results may be summarized the following way : the introduction of the solvency constraint lowers the nominal interest rate increase which is required to stabilize output and inflation in response to a financial asset price variation or an exchange rate depreciation which would increase aggregate demand. For a given price perception $E_t \pi_t$ determined by monetary policy, any increase in the nominal interest rate implies a decrease of g_t to meet the solvency constraint. Any restriction of monetary policy thus implies a corresponding restriction of fiscal policy. Therefore the necessary upward reaction of interest rates to positive shocks on stock prices or negative shocks on exchange rate is lower than what would be required in the absence of such a fiscal policy adjustment.

Technically, the solutions for L_t , e_t and R_t are the same as those given by eqs. (4.10), (4.11), (4.12), where β must be replaced with $\beta + \frac{X_{t-1}}{\bar{Y}}$. In particular, it is clear that replacing β with $\beta + \frac{X_{t-1}}{\bar{Y}}$ lowers the coefficients of ε_t^l and ε_t^x in

⁹It has to be pointed out that with this strong form of solvency constraint, a positive primary deficit is only allowed if inflation exceeds the nominal interest rate, thus if the real nominal interest rate is negative. A stationary long term equilibrium is only conceivable with an absence of primary deficit and a null real interest rate. This equilibrium value of the real interest rate is compatible with the absence of real growth in the model.

the interest rate solution.

6. Conclusion

In this paper we have clarified some of the issues about the relations between financial asset prices and monetary policy. The analysis of the former sections has led us to understand how monetary policy has to react to exogenous financial shocks and how the conduct of monetary policy has to take account of induced endogenous variations of financial asset prices. The complex interactions between interest rates, exchange rates and equity prices is summarised by the properties of a monetary conditions index which is derived from an optimising problem.

Several lessons may be drawn from our analysis :

- When monetary policy takes account of the role played by financial prices in the monetary transmission mechanism the optimal monetary policy reaction to a demand shock is inferior to what it would have been otherwise. This reduction of the magnitude of the interest rates reaction increases with the impact of financial prices on aggregate demand.

- This feature of the solutions has interesting implications since there are good reasons to assume that the impacts of financial prices on aggregate demand change over time. For example it may be suspected that the sensibility of consumer demand with respect to the value of equities has recently increased as the result of increasing stock holding by households. Therefore γ should have increased. An important consequence of this structural change is that the optimal monetary policy reaction to demand shocks should have decreased. This argument would imply a reduction of the volatility of intervention interest rates for a given level of the variance of aggregate demand.

- In an inflation targeting framework, monetary policy must react to unanticipated changes in asset prices only to the extent that they affect the output gap and thus the inflation outlook.

- To correctly assess the optimal response of interest rates to exogenous changes of the stochastic risk premia on equity prices and exchange rates, it is necessary to include both kinds of financial prices in the model because of their interaction.

- The optimal monetary policy response to financial shocks also depends on the assumed functioning of the economy and we have shown that a public debt solvency constraint lowers the interest rate response to financial shocks.

7. References

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8. Appendix 1: Derivation of linear approximation (2.6)

The wedge between real output and real wage costs is given by $Y - WN$, where Y is real output, W is the real wage and N is total employment. We simply assume that employment is proportional to output: $N = kY$ and the flow of real dividends becomes : $DIV = (1 - kW)Y$. In logarithms, we obtain $\ln(DIV) = \ln(1 - kW) + \ln(Y)$. The expression $\ln(1 - kW)$ is also equal to $\ln(1 - ke^{\ln(W)})$ and can be approximated around an arbitrary value $\ln(\bar{W})$ of $\ln(W)$ by the formula $\ln(1 - ke^{\ln(\bar{W})}) - (\frac{ke^{\ln(\bar{W})}}{1 - ke^{\ln(\bar{W})}})(\ln(W) - \ln(\bar{W}))$. We thus obtain the approximation of equation (2.6) with $\tau = -\ln(1 - ke^{\ln(\bar{W})}) - (\frac{ke^{\ln(\bar{W})}}{1 - ke^{\ln(\bar{W})}})\ln(\bar{W})$ and $\lambda = \frac{ke^{\ln(\bar{W})}}{1 - ke^{\ln(\bar{W})}} = \frac{k\bar{W}}{1 - k\bar{W}}$. When the real wage level \bar{W} around which we realize the approximation is equal to 1, λ is equal to $\frac{k}{1-k}$ and when k is small, τ is roughly equal to k which is the inverse labour productivity rate.

9. Appendix 2: Deriving solutions (4.10) and (4.11)

Eqs. 4.8 and 4.9 may be rewritten as :

$$\begin{aligned} L_t = & a + bE_t L_{t+1} + cE_t e_{t+1} + d\varepsilon_t^x + mE_t(\varepsilon_t^d - \varepsilon_t^s) + nE_t \varepsilon_t^s + pE_t \varepsilon_t^w + v\varepsilon_t^l \\ & + wp_{t-1} + q(lw_{t-1} - p_{t-1}) + i\eta_{t-1} + jE_t p_t^* \end{aligned} \quad (9.1)$$

where

$$\begin{aligned} a = & \left(\frac{(\beta + \mu)((1 - \rho)(\bar{y} - \tau)) - \alpha + \bar{y}}{\beta + \gamma + \mu} \right) - \left(\frac{\mu}{\beta + \gamma + \mu} \right) R^* \\ & + \left(\frac{2(\beta + \mu)(1 - \rho) - (\beta - \gamma - \mu)}{\beta + \gamma + \mu} \right) \bar{\pi} \\ b = & \frac{(\beta + \mu)\rho}{\beta + \gamma + \mu}; c = \left(\frac{\mu}{\beta + \gamma + \mu} \right); d = - \left(\frac{\mu}{\beta + \gamma + \mu} \right); m = - \left(\frac{1}{\beta + \gamma + \mu} \right); \end{aligned}$$

$$\begin{aligned}
n &= \left(\frac{(\beta + \mu)(1 - \rho)}{\beta + \gamma + \mu} \right); p = - \left(\frac{(\beta + \mu)(1 - \rho)\lambda}{\beta + \gamma + \mu} \right); v = - \left(\frac{(\beta + \mu)}{\beta + \gamma + \mu} \right); \\
q &= \left(\frac{-(1 - \rho)\lambda(\beta + \mu)}{\beta + \gamma + \mu} \right); i = \left(\frac{\mu(\mu + \gamma - \theta\beta\chi') - (1 - \rho)(1 + \lambda - \theta\chi')\beta(\beta + \mu)}{\beta(\beta + \gamma + \mu)(1 + \chi')} \right); \\
j &= \frac{-\mu}{\beta + \gamma + \mu}; w = \left(\frac{(\beta + \mu)(1 - \rho) + (\gamma + \mu)}{\beta + \gamma + \mu} \right)
\end{aligned} \tag{9.2}$$

and

$$\begin{aligned}
e_t &= A + BE_t L_{t+1} + CE_t e_{t+1} + D\varepsilon_t^x + ME_t(\varepsilon_t^d - \varepsilon_t^s) + NE_t \varepsilon_t^s + PE_t \varepsilon_t^w + V\varepsilon_t^l \\
&\quad + Wp_{t-1} + Q(lw_{t-1} - p_{t-1}) + I\eta_{t-1} + JE_t p_t^*
\end{aligned} \tag{9.3}$$

where

$$\begin{aligned}
A &= \left(\frac{2\gamma(1 - \rho) + (\beta - \gamma - \mu)}{\beta + \gamma + \mu} \right) \bar{\pi} - \left(\frac{\beta + \gamma}{\beta + \gamma + \mu} \right) R^* + \left(\frac{\alpha - \bar{y} + \gamma(1 - \rho)(\bar{y} - \tau)}{\beta + \gamma + \mu} \right) \\
B &= \left(\frac{\gamma\rho}{\beta + \gamma + \mu} \right); C = \left(\frac{\beta + \gamma}{\beta + \gamma + \mu} \right); D = \left(\frac{-(\gamma + \beta)}{\beta + \gamma + \mu} \right); M = \left(\frac{1}{\beta + \gamma + \mu} \right); \\
N &= \left(\frac{\gamma(1 - \rho)}{\beta + \gamma + \mu} \right); P = \left(\frac{-\gamma(1 - \rho)\lambda}{\beta + \gamma + \mu} \right); V = \left(\frac{-\gamma}{\beta + \gamma + \mu} \right); \\
Q &= - \left(\frac{\gamma(1 - \rho)\lambda}{\beta + \gamma + \mu} \right); I = \frac{(\beta + \gamma)(\mu + \gamma - \theta\chi') - \gamma(1 - \rho)(1 + \lambda - \theta\chi')\beta}{(\beta + \gamma + \mu)\beta(1 + \chi')}; \\
J &= \left(\frac{\mu}{\beta + \gamma + \mu} \right); W = - \left(\frac{(\gamma\rho + \mu)}{\beta + \gamma + \mu} \right)
\end{aligned} \tag{9.4}$$

It is interesting to note that :

$$\begin{aligned}
b + B &= \rho; c + C = 1; d + D = 1; m + M = 0; n + N = 1 - \rho; p + P = (1 - \rho)\lambda; \\
v + V &= -1; w + W = 1 - \rho; nB = Nb
\end{aligned} \tag{9.5}$$

Eq. (9.1) implies that :

$$\begin{aligned}
E_t e_{t+1} &= (L_t - a - bE_t L_{t+1} - d\varepsilon_t^x - mE_t(\varepsilon_t^d - \varepsilon_t^s) - nE_t \varepsilon_t^s - pE_t \varepsilon_t^w - v\varepsilon_t^l \\
&\quad - wp_{t-1} - q(lw_{t-1} - p_{t-1}) - i\eta_{t-1} - jE_t p_t^*)/c
\end{aligned} \tag{9.6}$$

which can be substituted into eq.(9.3) to yield

$$\begin{aligned}
e_t = & \left(A - \frac{aC}{c}\right) + \left(B - \frac{bC}{c}\right)E_t L_{t+1} + \left(D - \frac{dC}{c}\right)\varepsilon_t^x \\
& + \left(M - \frac{mC}{c}\right)E_t(\varepsilon_t^d - \varepsilon_t^s) + \left(N - \frac{nC}{c}\right)E_t \varepsilon_t^s \\
& + \left(P - \frac{pC}{c}\right)E_t \varepsilon_t^w + \left(V - \frac{vC}{c}\right)\varepsilon_t^l + \frac{C}{c}L_t \\
& + \left(W - w\frac{C}{c}\right)p_{t-1} + \left(Q - q\frac{C}{c}\right)(lw_{t-1} - p_{t-1}) \\
& + \left(I - i\frac{C}{c}\right)\eta_{t-1} + \left(J - j\frac{C}{c}\right)E_t p_t^*
\end{aligned} \tag{9.7}$$

Taking the expectations on date t-1 of both sides of this equation we obtain :

$$\begin{aligned}
E_{t-1}e_t = & \left(A - \frac{aC}{c}\right) + \left(B - \frac{bC}{c}\right)E_{t-1}L_{t+1} + \left(D - \frac{dC}{c}\right)E_{t-1}\varepsilon_t^x \\
& + \left(M - \frac{mC}{c}\right)E_{t-1}(\varepsilon_t^d - \varepsilon_t^s) + \left(N - \frac{nC}{c}\right)E_{t-1}\varepsilon_t^s \\
& + \left(P - \frac{pC}{c}\right)E_{t-1}\varepsilon_t^w + \left(V - \frac{vC}{c}\right)E_{t-1}\varepsilon_t^l + \frac{C}{c}E_{t-1}L_t \\
& + \left(W - w\frac{C}{c}\right)E_{t-1}p_{t-1} + \left(Q - q\frac{C}{c}\right)E_{t-1}(lw_{t-1} - p_{t-1}) \\
& + \left(I - i\frac{C}{c}\right)E_{t-1}\eta_{t-1} + \left(J - j\frac{C}{c}\right)E_{t-1}p_t^*
\end{aligned} \tag{9.8}$$

Lagging eq. (9.6) one period yields :

$$\begin{aligned}
E_{t-1}e_t = & (L_{t-1} - a - bE_{t-1}L_t - d\varepsilon_{t-1}^x - mE_{t-1}(\varepsilon_{t-1}^d - \varepsilon_{t-1}^s) - nE_{t-1}\varepsilon_{t-1}^s \\
& - pE_{t-1}\varepsilon_{t-1}^w - v\varepsilon_{t-1}^l - wp_{t-2} - q(lw_{t-2} - p_{t-2}) - i\eta_{t-2} \\
& - jE_{t-1}p_{t-1}^*)/c
\end{aligned} \tag{9.9}$$

Eqs. (9.8) and (9.9) combine to yield :

$$\begin{aligned}
L_{t-1} = & (a + Ac - aC) + (b + C)E_{t-1}L_t + (Bc - bC)E_{t-1}L_{t+1} + d\varepsilon_{t-1}^x \\
& + (cD - Cd)E_{t-1}\varepsilon_t^x + nE_{t-1}\varepsilon_{t-1}^s + (cN - nC)E_{t-1}\varepsilon_t^s \\
& + pE_{t-1}\varepsilon_{t-1}^w + (cP - pC)E_{t-1}\varepsilon_t^w + v\varepsilon_{t-1}^l + (cV - vC)E_{t-1}\varepsilon_t^l
\end{aligned}$$

$$\begin{aligned}
& +mE_{t-1}(\varepsilon_{t-1}^d - \varepsilon_{t-1}^s) + (cM - mC)E_{t-1}(\varepsilon_t^d - \varepsilon_t^s) \\
& +wp_{t-2} + (cW - wC)E_{t-1}p_{t-1} + i\eta_{t-2} + (cI - iC)E_{t-1}\eta_{t-1} \\
& +jE_{t-1}p_{t-1}^* + (cJ - jC)E_{t-1}p_t^* + q(lw_{t-2} - p_{t-2}) \\
& + (cQ - qC)E_{t-1}(lw_{t-1} - p_{t-1})
\end{aligned} \tag{9.10}$$

Leading this equation one period we obtain

$$\begin{aligned}
L_t = & (a + Ac - aC) + (b + C)E_tL_{t+1} + (Bc - bC)E_tL_{t+2} \\
& +d\varepsilon_t^x + (cD - Cd)E_t\varepsilon_{t+1}^x + nE_t\varepsilon_t^s + (cN - nC)E_t\varepsilon_{t+1}^s \\
& +pE_t\varepsilon_t^w + (cP - pC)E_t\varepsilon_{t+1}^w + v\varepsilon_t^l + (cV - vC)E_t\varepsilon_{t+1}^l \\
& +mE_t(\varepsilon_t^d - \varepsilon_t^s) + (cM - mC)E_t(\varepsilon_{t+1}^d - \varepsilon_{t+1}^s) \\
& +wp_{t-1} + (cW - wC)E_t p_t + i\eta_{t-1} + (cI - iC)E_t\eta_t \\
& +jE_t p_t^* + (cJ - jC)E_t p_{t+1}^* + q(lw_{t-1} - p_{t-1}) \\
& + (cQ - qC)E_t(lw_t - p_t)
\end{aligned} \tag{9.11}$$

This equation is a second order linear difference equation. To solve it forward, we rewrite (9.1) using the lag operator \mathcal{L} :

$$\begin{aligned}
& E_t \left((1 - (b + C)\mathcal{L}^{-1} - (Bc - bC)\mathcal{L}^{-2})L_t \right) \\
= & E_t \left(\begin{aligned} & (a + Ac - aC) + d\varepsilon + (cD - Cd)\varepsilon_{t+1}^x + n\varepsilon_t^s \\ & + (cN - nC)\varepsilon_{t+1}^s + p\varepsilon_t^w + (cP - pC)\varepsilon_{t+1}^w + v\varepsilon_t^l \\ & + (cV - vC)\varepsilon_{t+1}^l + m(\varepsilon_t^d - \varepsilon_t^s) \\ & + (cM - mC)(\varepsilon_{t+1}^d - \varepsilon_{t+1}^s) + wp_{t-1} \\ & + (cW - wC)p_t + i\eta_{t-1} + (cI - iC)\eta_t \\ & + jp_t^* + (cJ - jC)p_{t+1}^* \\ & + q(lw_{t-1} - p_{t-1}) + (cQ - qC)(lw_t - p_t) \end{aligned} \right)
\end{aligned} \tag{9.12}$$

Dynamic stability requires that $(b + C) + (Bc - bC) < 1$, $(b + C) - (Bc - bC) > 1$ and $|(Bc - bC)| < 1$. Substituting from 9.2 and 9.4 these conditions reduce to $\frac{\beta + \gamma + \mu\rho}{\beta + \gamma + \mu} < 1$, and $\frac{\beta(2\rho + 1) + \mu\rho + \gamma}{\beta + \gamma + \mu} > -1$, $\left| -\frac{\beta\rho}{\beta + \gamma + \mu} \right| < 1$. These conditions are satisfied under the reasonable hypotheses that $\beta > 0$, $\gamma > 0$, $\mu > 0$ and $0 < \rho < 1$. Dividing by the second order polynomial in the lag operator, we obtain :

$$E_t L_t = L_t = E_t \left(\begin{pmatrix} 1 - ((b+C)\mathcal{L}^{-1} - (Bc-bC)\mathcal{L}^{-2})^{-1} \\ (a+Ac-aC) + d\varepsilon + (cD-Cd)\varepsilon_{t+1}^x \\ + n\varepsilon_t^s + (cN-nC)\varepsilon_{t+1}^s + p\varepsilon_t^w \\ + (cP-pC)\varepsilon_{t+1}^w + v\varepsilon_t^l + (cV-vC)\varepsilon_{t+1}^l \\ + m(\varepsilon_t^d - \varepsilon_t^s) + (cM-mC)(\varepsilon_{t+1}^d - \varepsilon_{t+1}^s) \\ + wp_{t-1} + (cW-wC)p_t + i\eta_{t-1} \\ + (cI-iC)\eta_t + jp_t^* + (cJ-jC)p_{t+1}^* \\ + q(lw_{t-1} - p_{t-1}) + (cQ-qC)(lw_t - p_t) \end{pmatrix} \right) \quad (9.13)$$

This equation is equivalent to

$$L_t = \sum_{i=0}^{\infty} E_t \left(\begin{pmatrix} ((b+C)\mathcal{L}^{-1} + (Bc-bC)\mathcal{L}^{-2})^i \\ (a+Ac-aC) + d\varepsilon_t^x + (cD-Cd)\varepsilon_{t+1}^x \\ + n\varepsilon_t^s + (cN-nC)\varepsilon_{t+1}^s + p\varepsilon_t^w \\ + (cP-pC)\varepsilon_{t+1}^w + v\varepsilon_t^l + (cV-vC)\varepsilon_{t+1}^l \\ + m(\varepsilon_t^d - \varepsilon_t^s) + (cM-mC)(\varepsilon_{t+1}^d - \varepsilon_{t+1}^s) \\ + wp_{t-1} + (cW-wC)p_t + i\eta_{t-1} \\ + (cI-iC)\eta_t + jp_t^* + (cJ-jC)p_{t+1}^* \\ + q(lw_{t-1} - p_{t-1}) + (cQ-qC)(lw_t - p_t) \end{pmatrix} \right) \quad (9.14)$$

which reduces to

$$\begin{aligned} L_t = & \frac{a+Ac-aC}{1-(b+C)-(Bc-bC)} + \frac{(w+cW-wC)p_{t-1}}{1-(b+C)-(Bc-bC)} \\ & + \left(\frac{(w+cW-wC) \left(\frac{(b+C)+2(Bc-bC)}{(1-(b+C)-(Bc-bC))^2} \right)}{+ \frac{cW-wC}{1-(b+C)-(Bc-bC)}} \right) \bar{\pi} \\ & + \frac{(q+cQ-qC)}{1-(b+C)-(Bc-bC)} (lw_{t-1} - p_{t-1}) \\ & + \frac{(j+cJ-jC)}{1-(b+C)-(Bc-bC)} p_t^* \\ & + \left(\frac{(j+cJ-jC) \left(\frac{(b+C)+2(Bc-bC)}{(1-(b+C)-(Bc-bC))^2} \right)}{+ \frac{cJ-jC}{1-(b+C)-(Bc-bC)}} \right) \bar{\pi}^* \\ & + \left(i + \frac{q(b+C) + (cQ-qC)}{1+\chi'} - \frac{w(b+C) + (cW-wC)}{1+\chi'} \right) \eta_{t-1} \end{aligned}$$

$$\begin{aligned}
& +d\varepsilon_t^x + p\varepsilon_t^w + v\varepsilon_t^l \\
& + \frac{(m + (cM - mC)\delta)\delta\varepsilon_{t-1}^d}{1 - (b + C)\delta - (Bc - bC)\delta^2} \\
& - \frac{(m + cM - mC - n - cN + nC)\varepsilon_{t-1}^s}{1 - (b + C) - (Bc - bC)}
\end{aligned} \tag{9.15}$$

Following the same procedure we obtain a second order linear difference equation for e_t :

$$\begin{aligned}
e_t = & (A + aB - Ab) + (b + C)E_t e_{t+1} + (Bc - bC)E_t e_{t+2} \\
& + D\varepsilon_t^x + (Bd - bD)E_t \varepsilon_{t+1}^x + NE_t \varepsilon_t^s + (nB - Nb)E_t \varepsilon_{t+1}^s \\
& + PE_t \varepsilon_t^w + (pB - Pb)E_t \varepsilon_{t+1}^w + V\varepsilon_t^l + (vB - Vb)E_t \varepsilon_{t+1}^l \\
& + ME_t(\varepsilon_t^d - \varepsilon_t^s) + (mB - Mb)E_t(\varepsilon_{t+1}^d - \varepsilon_{t+1}^s) \\
& + Wp_{t-1} + (wB - Wb)E_t p_t + I\eta_{t-1} + (iB - Ib)E_t \eta_t \\
& + JE_t p_t^* + (jB - Jb)E_t p_{t+1}^* + Q(lw_{t-1} - p_{t-1}) \\
& + (qB - Qb)E_t(lw_t - p_t)
\end{aligned} \tag{9.16}$$

the solution of which is

$$\begin{aligned}
e_t = & \frac{A + aB - Ab}{1 - (b + C) - (Bc - bC)} + \frac{(W + wB - Wb)p_{t-1}}{1 - (b + C) - (Bc - bC)} \\
& + \left(\frac{(W + wB - Wb) \left(\frac{(b+C)+2(Bc-bC)}{(1-(b+C)-(Bc-bC))^2} \right)}{\frac{wB-Wb}{1-(b+C)-(Bc-bC)}} \right) \bar{\pi} \\
& + \frac{(Q + qB - Qb)}{1 - (b + C) - (Bc - bC)}(lw_{t-1} - p_{t-1}) \\
& + \frac{(J + jB - Jb)}{1 - (b + C) - (Bc - bC)}p_t^* \\
& + \left(\frac{(J + jB - Jb) \left(\frac{(b+C)+2(Bc-bC)}{(1-(b+C)-(Bc-bC))^2} \right)}{\frac{jB-Jb}{1-(b+C)-(Bc-bC)}} \right) \bar{\pi}^* \\
& + \left(I + \frac{Q(b + C) + (qB - Qb)}{1 + \chi'} - \frac{W(b + C) + (wB - Wb)}{1 + \chi'} \right) \eta_{t-1} \\
& + D\varepsilon_t^x + P\varepsilon_t^w + V\varepsilon_t^l
\end{aligned}$$

$$\begin{aligned}
& + \frac{(M + (mB - Mb)\delta)\delta\varepsilon_{t-1}^d}{1 - (b + C)\delta - (Bc - bC)\delta^2} \\
& - \frac{(M + mB - Mb - N - nB + Nb)\varepsilon_{t-1}^s}{1 - (b + C) - (Bc - bC)}
\end{aligned} \tag{9.17}$$

Substituting eqs. (9.2) and (9.4) into eqs. (9.15) and (9.17) we obtain solutions (4.10) and (4.11) in the main text. To derive these solutions, we have used some important properties of the model :

$$\begin{aligned}
E_t p_t &= p_{t-1} + \bar{\pi} - \eta_{t-1}; E_t p_{t+i} = p_{t-1} + i\bar{\pi}, \forall i > 0; \\
E_t p_{t+i}^* &= p_t^* + i\bar{\pi}^*, \forall i \geq 0; E_t(w_t - p_t) = w_{t-1} - p_{t-1} + \eta_{t-1};
\end{aligned} \tag{9.18}$$

$$E_t(w_{t+i} - p_{t+i}) = w_{t-1} - p_{t-1}, \forall i > 0; E_t\varepsilon_{t+i}^d = \delta^{i+1}\varepsilon_{t-1}^d, \forall i \geq 0; \tag{9.19}$$

$$E_t\varepsilon_{t+i}^s = \varepsilon_{t-1}^s, \forall i \geq 0; E_t\varepsilon_{t+i}^l = E_t\varepsilon_{t+i}^x = E_t\varepsilon_{t+i}^w = 0, \forall i > 0 \tag{9.20}$$

10. Appendix 3: Derivation of the fiscal solvency constraint

We represent the nominal public debt with LX . The dynamics of this variable are given by

$$LX_t = (LG_t + LX_{t-1})(1 + R_t) \tag{10.1}$$

where LG_t is the nominal primary public deficit. Dividing both sides of the latter equation by P_t , we obtain

$$X_t = G_t(1 + R_t) + X_{t-1} \left(\frac{(1 + R_t)}{(P_t - P_{t-1})} \right) \tag{10.2}$$

or

$$X_t = G_t(1 + R_t) + X_{t-1} \left(\frac{(1 + R_t)}{(1 + \pi_t)} \right) \tag{10.3}$$

where the real primary deficit $G_t = \frac{LG_t}{P_t}$, while $X_t = \frac{LX_t}{P_t}$ and $X_{t-1} = \frac{LX_{t-1}}{P_{t-1}}$ (X represents the real public debt level).

Imposing $X_t = X_{t-1}$, it implies that

$$\pi_t - R_t = \left(\frac{G_t(1 + R_t)(1 + \pi_t)}{X_{t-1}} \right) \tag{10.4}$$

$$\frac{G_t}{X_{t-1}} = \frac{(\pi_t - R_t)}{(1 + R_t)(1 + \pi_t)} \quad (10.5)$$

The Taylor first order approximation of the equation (10.5), around the null value for π_t and R_t , is equal to $\pi_t - R_t$. By definition G_t is equal to $g_t \bar{Y}$. Therefore, we obtain the equation (5.2).